



### ANDREI ZELEVINSKI

Andrei Zelevinski, a brilliant representative of the Moscow mathematical school, passed away on April 10, 2013 at the age of sixty. He was a great mathematician, a kind and good man. He liked to teach, and taught all his life. Andrei was one of the organizers of the People's Jewish University in the early 80's when Moscow mathematicians wanted to help gifted students rejected by the Moscow State University because of their origin or for political reasons. Andrei thought a lot on how to teach, how to put together the "traditional" and "modern" parts of mathematics. He was among those whose activity led to the foundation of the Independent University of Moscow, which follows many of his ideas.

Our love and memory stay with him.

*B. Feigin, S. Khoroshkin, S. Lando, A. Sossinsky, M. Tsfasman*

## MY DEAR FRIEND ANDREI ZELEVINSKI

I have known Andrei for a long time, from our childhood. In fact, our parents knew each other. We met about forty-seven years ago, maybe more, and we became friends. As a small boy I had dreamed of, had eagerly longed for a unique and exceptional friend. I chose Andrei. We were students of the same class, 7a, in the famous mathematical school #2. It was an interesting place, where lots of bright boys and girls studied. The teachers were exceptional, especially in literature. Life there was restless and colorful, but at the same time filled with high tension.

Many interesting persons surrounded me, but I had reasons to choose Andrei as a mentor, the object of my youthful love and admiration. In our class he was tops in mathematics, and probably he had the strongest mind. He was also very adept at solving mathematical olympiad problems. But all this was not the main point. I lived in a state of a permanent diffidence, perhaps something normal for a boy of my nature at twelve. Andrei was very different. In him, one could feel inner strength, a solid core. Surely, he was not a superhero. He had his own weak points, but my impression is that he had somehow found the solution of the main problem of our lives, namely how to live “in a right way”, how to live with dignity. I think the other students from our class had similar feelings about him. Andrei was the leader of the small group of comrades in our school.

Andrei managed to become one of the best mathematicians of his generation. He entered the mathematical department of Moscow University, where he chose I. M. Gelfand as a teacher, and I did as well. Izrael Moiseevich Gelfand was an extraordinary person from many (maybe all) points of view. There was a sort of permanent swirl around him. He managed to organize a great mathematical community, maybe the greatest in the world. We became the members of the Gelfand school, so I was a witness to the first steps of Andrei in mathematics. Four boys from our class, Andrei, Sasha Kamensky, Yura Litvin, and I, became Gelfand’s students. We attended the Gelfand seminar each Monday in room 1408 of the main building of Moscow University.

Gelfand had a collection of archetypical problems for freshmen, so-called “brain teasers”. One of them was the question about the classification of triples of subspaces in a vector space. It was a test in understanding linear algebra. This problem was very educational. We all solved it, but only Andrei took it seriously. He needed a starting point, wanted to do real mathematics. He was persistent, and after some time wrote his first paper about the algebra of intertwiners in the category of representations of the general linear group over a finite field.

Gelfand taught his students by rather nontraditional methods. Very rarely did he put some specific problem in front of us. We lived like fish in the sea with an abundance of food around. There was a lot of approachable problems, a lot of possibilities... But nobody put food in one’s mouth. You had to find your way by

yourself, with some help from others. Andrei got support from Joseph Bernstein, a very generous mathematician and teacher. Joseph also was a student of Gelfand, and one of the central figures in his seminar. Communication with Gelfand was not an easy task; with Joseph, it was much easier. Andrei and Joseph became friends. The two collaborated in writing several remarkable papers.

The life of a mathematician follows a predictable pattern. At every moment, you have some question in mind, usually many questions. You want to find the answer, or at least to take a small step forward. This appears vital for the continuation of your life. And you fail. After some depression, you start again, fail and start again... Only very rarely do you advance. But there is an additional consideration that adds gravity to your pursuit. You know there are other mathematicians obsessed by the same or similar problems, some of them definitely smarter than yourself. Therefore, every person who has managed to become mathematician needs to invent some way to sustain such a life and not to go crazy. We are also all human beings. We need friendship and love, so we have wives and children... As a result, for mathematicians, the nearly unsolvable problem is how to reconcile in a relatively reasonable way all these contradictions, and to find some small amount of inner peace and happiness. An unsolvable problem, yes, but if you want to be mathematician, you need to find a solution.

I think Andrei found one. I remember that rather early in life he had formulated ideas concerning what kind of wife he wanted and how to organize his family. Usually, if a young person formulates such ideas about his future life, it may seem strange. But not so in Andrei's case. He was very serious and I understood he had the necessary clarity and sense of purpose.

Similarly, he planned his mathematical future. When just a schoolboy, he could already formulate what kind of mathematical problems he was drawn to and to which he wanted to devote his life. His favorite subject was combinatorics. He planned to become a specialist in combinatorics, while in reality he worked in the field of representation theory. His style was very combinatorial, in a good sense. This means that he always worked on carefully chosen concrete problems, where he could do many things by hand. The famous cluster algebras appeared in by this way. Many times in his life, Andrei returned to the question of constructing an explicit base in the representations of Lie algebras, groups, quantum groups... In particular, he wanted to understand the famous Lusztig and Kashiwara canonical base in representations of quantum groups. The constructions of Lusztig and Kashiwara were not very explicit. Andrei wanted to find a more manageable approach. In order to describe vectors of the base by some formula or algorithm, he discovered the remarkable transformation formula. This was the starting point for his definition of cluster algebras. We can see a similar approach in all his works, for example in his seminal papers (with J. N. Bernstein) on the representations of  $p$ -adic groups. I like his papers about the representations of the general linear group over finite fields, where he uses machinery of Hopf algebras. Among other things, in these articles he found new approaches to many well-known combinatorial problems.

By the way, the talent and style of Andrei can be compared with that of one of the best mathematicians of our century, Ian Macdonald. He, like Andrei, was a great specialist in representation theory, but often started from combinatorics and,

after some wandering, returned to combinatorics. So, it was natural that Andrei translated Macdonald's famous book into Russian and added at the end of that book his own results in combinatorics.

I was friends with Andrei in our school years, and was close to him later. Unfortunately, our relationship almost stopped after his emigration to America. But he remains one of the most important persons in my life. I still love him.

*Boris Feigin*